UNIVERSITY OF TEXAS AT AUSTIN
Dept. of Electrical and Computer Engineering

Quiz #2

Date: November 6, 2003

Course: EE 313 Evans/Arifler

Name: SOLUTIONS
Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework and solution sets.
- You may find the following sections/tables in Lathi useful:
  - Section B.10-3 Power Series (p. 64)
  - Section B.10-6 Trigonometric Identities (p. 65)
  - Table 4.1 Laplace Transform Pairs (p. 256)
  - Table 4.2 Laplace Transform Operations (p. 273)
  - Table 5.1 Z-Transform Pairs (p. 359)
  - Table 5.2 Z-Transform Operations (p. 367)
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- Fully justify your answers unless instructed otherwise.

Problem	Point Value	Your Score	Topic
1	20		Laplace Transforms
2	20		Z-Transforms
3	20		Difference Equation
4	20		Frequency Response
5	20		Miscellaneous
Total	100		

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Problem 2.2 Z-Transforms. 20 points.

For this problem, let us define the Z-transform of $f[k]$ as follows:

$$F[z] \equiv \sum_{k=-\infty}^{\infty} f[k] z^k \tag{1}$$

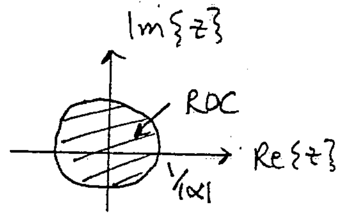
(Note that $f[k]$ is being multiplied by z^k instead of z^{-k}.)

(a) (10 points) Compute the Z-transform of $f[k] = \alpha^k u[k]$ using the Z-transform definition given in (1). State and sketch the region of convergence (ROC). In your sketch, clearly label the axes, axis intercepts, and mark the ROC.

$$F[z] = \sum_{k=-\infty}^{\infty} \alpha^k u[k] z^k = \sum_{k=0}^{\infty} \alpha^k z^k = \sum_{k=0}^{\infty} (\alpha z)^k$$

$$= \frac{1}{1-\alpha z}, \quad |\alpha z| < 1$$

ROC : $|z| < \frac{1}{|\alpha|}$



(b) (10 points) Compute the Z-transform of $f[k] = -\alpha^k u[-k-1]$ using the Z-transform definition given in (1). State and sketch the region of convergence (ROC). In your sketch, clearly label the axes, axis intercepts, and mark the ROC.

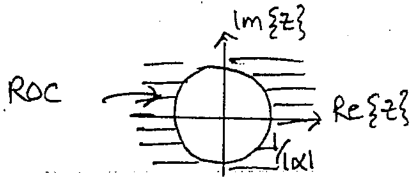
$$F[z] = \sum_{k=-\infty}^{\infty} -\alpha^k u[-k-1] z^k = \sum_{k=-\infty}^{-1} -\alpha^k z^k$$

$$= -\sum_{k=-\infty}^{-1} (\alpha z)^k = -\left(\frac{1}{\alpha z} + \frac{1}{(\alpha z)^2} + \frac{1}{(\alpha z)^3} + \cdots\right)$$

$$= 1 - \left(1 + \frac{1}{\alpha z} + \frac{1}{(\alpha z)^2} + \frac{1}{(\alpha z)^3} + \cdots\right.$$

ROC : $|z| > \frac{1}{|\alpha|}$

$$= 1 - \frac{1}{1 - \frac{1}{\alpha z}}, \quad \left|\frac{1}{\alpha z}\right| < 1$$



$$= 1 - \frac{\alpha z}{\alpha z - 1} = \frac{-1}{\alpha z - 1} = \frac{1}{1-\alpha z}$$

Problem 2.4 Frequency Response. 20 points.

(a) (10 points) Determine the transfer function and (causal) impulse response of the filter described by

$$y[k] = 0.25y[k-2] + f[k],$$

where $f[k]$ is the input and $y[k]$ is the output.

$$Y[z] = 0.25 \frac{Y[z]}{z^2} + F[z]$$

$$Y[z]\left(1 - \frac{0.25}{z^2}\right) = F[z] \Rightarrow H[z] = \frac{1}{1-0.25z^{-2}}$$

$$H[z] = \frac{A}{1-0.5z^{-1}} + \frac{B}{1+0.5z^{-1}} = \frac{0.5}{1-0.5z^{-1}} + \frac{+0.5}{1+0.5z^{-1}}$$

$$h[k] = \mathcal{Z}^{-1}\{H[z]\} = 0.5(0.5)^k u[k] + 0.5(-0.5)^k u[k]$$

(b) (10 points) Based on the transfer function you determined in part (a), find $|H[e^{j\Omega}]|$ (the amplitude response) when

(i) $\Omega = 0$.

$$|H[e^{j0}]| = |H[\cos 0 + j\sin 0]| = |H[1]| = \frac{1}{1-0.25(1)^{-2}}$$

$$= \frac{1}{0.75} = 4/3$$

(ii) $\Omega = \pi/2$.

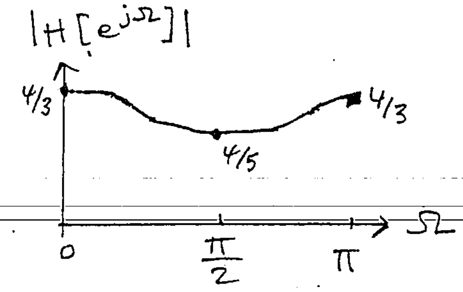
$$|H[e^{j\pi/2}]| = |H[\cos\tfrac{\pi}{2} + j\sin\tfrac{\pi}{2}]| = |H[j]| = \frac{1}{1-0.25(j)^{-2}}$$

$$= \frac{1}{1.25} = 4/5$$

(iii) $\Omega = \pi$.

$$|H[e^{j\pi}]| = |H[e^{j0}]| = 4/3$$

(iv) Using your answers for (i), (ii), and (iii), sketch the amplitude response for $0 \le \Omega \le \pi$. Comment on the shape of the amplitude response.



Filter will enhance low & high frequencies, attenuate intermediate frequencies.

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